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ERROR CONCEALMENT USING A DIFFUSION BASED METHOD

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ABSTRACT

In this paper, we present a novel PDE based error concealment algorithm. We formulate the error concealment problem as a sequential optimization problem with both smoothing and orientation constraints. By introducing the orientation constraint we convert a nonlinear variational problem into a problem that is well posed and which can be solved without iterative operations. A modified orientation diffusion scheme is presented which is able to reconstruct complex orientation patterns within blocks which have been lost in an image. In the intensity reconstruction stage which follows orientation diffusion, optimization is performed based on the orientation estimates from the first stage together with the constraint of smoothness on block boundaries. We present an efficient numerical scheme which implements the method without iterations.

1. INTRODUCTION

With block-based transform image coding schemes, for example JPEG, a one bit error can cause the decoding failure of a entire image block. Bit errors can also cause loss of synchronization and thus result in erroneous decoding of the following image blocks. Error concealment is an important technology to compensate for the loss of image blocks which cannot be well recovered by other error correcting methods. Various error concealment schemes have been presented in the literature. The error concealment scheme presented by Wang *et al.* [1] formulates the problem as an optimization problem to find a block matching the boundary that is maximally smooth. Smoothing constraints are applied to all pairs of samples in the directions of west, east, north and south. Blurred image blocks occur in higher frequency portions of an image, such as in edge regions. Sun and Kwok [2] present a POCS (Projections onto Convex Sets) model that can utilize spatial information more thoroughly by interpolation on a large local neighborhood of surrounding pixels. In the POCS iterative restoration process, a directional constraint, instead of only a smoothness constraint, is applied to the method of projections onto convex sets. They also present a multi-directional interpolation method [3]. Rabiee *et al.* [4] propose a multi-directional recursive nonlinear filtering (MRNF) scheme for spatial interpolation. Any missing block is reconstructed by a recursive process from the boundary to the center of the missing block. Lee *et al.* [5] employ fuzzy logic reasoning to recover high frequency information based on the fuzzy relationships between lost blocks and their neighbors. Zeng [6] presents a edge adapted error concealment scheme. In [7] an interpolation model is presented based on the continuity of level lines. This latter scheme has a lot of correlation with the proposed model. Li [8] presents a recursive model for error conealment with directional inference. In this paper, we focus on error concealment based on a PDE (partial differential equation) model. A traditional nonlinear model for error concealment is formulated as an optimization problem with a smoothing constraint on block boundaries which is based on a non-quadratic norm. Such norms have been shown to be able to conserve edges in image segmentation and smoothing problems, but this kind of model is often ill-posed. The problem of ill-posedness is not acceptable for error concealment since the solution relies strongly on the initial estimates of lost blocks. An alternate solution to the nonlinear model recursively estimates the pixels in a image block layer by layer from the boundary to the center. Since the nonlinear problem is ill-posed, the resulting reconstructed image depends on the sequence taken by this recursive process.

In this paper, we present a complete mathematical framework for PDE-based error concealment and its efficient numerical implementation. We formulate the error concealment problem as a sequential optimization process with both smoothing and orientation constraints. By introducing the constraint of orientation we convert a nonlinear variational problem into a problem that is well-posed and which can be solved without iterations. We use a more complex and systematic model than the traditional geometrical model for estimating the orientation information in lost blocks based on the orientation diffusion model, which can reconstruct

complex orientation patterns with more accuracy within lost blocks. In the subsequent intensity reconstruction stage, optimization based on the constraint of smoothness on the boundary is performed; this stage uses the orientation estimates obtained in the first stage.

2. ERROR CONCEALMENT BASED ON SEQUENTIAL PDE

The error concealment problem is solved by a sequential optimization scheme involving the following two stages.

2.1. Stage 1: Orientation diffusion

In the first stage, optimization is performed to estimate image orientation such that it is maximally smooth in the reconstructed areas. This problem can be formulated as,

$$\widehat{\theta} = \min_{\theta}(C_{\theta}[\theta(x,y)])$$

with the constraint,

$$\theta(x,y) = \theta'(x,y) \qquad (x,y) \in B$$

where $\theta = \frac{\pi}{2} + \arg[\bigtriangledown u(x,y)]$ is the orientation for pixel $u(x,y)$; $\theta'(x,y)$ is the correctly decoded orientation on lost block boundaries; $C_{\theta}[.]$ is a non-negative monotonic cost function; and B is the set of boundary pixels of lost blocks.

The orientation at a pixel is defined as the direction that is orthogonal to the gradient direction at the pixel. The scheme of orientation estimation in this paper is inspired by the orientation diffusion for noisy image data presented by Perona [9]. We have adapted Perona's scheme to the problem of error concealment and have made an adjustment to it, by adding a weighting $w(x,y)$ in the formulation of the continuous domain orientation diffusion problem:

$$\widehat{\theta} = \min_{\theta}(\int_x \int_y w(x,y)[1 - \cos(|\bigtriangledown \theta(x,y)|)]dxdy). \quad (1)$$

With $w(x,y)$ proportional to $|\bigtriangledown u(x,y)|$, weak gradient pixels are no longer unduly influential in the estimation of the orientations of neighbouring pixels.

The Euler Equation corresponding to (1) is,

$$\bigtriangledown \cdot [\frac{w(x,y)\sin(|\bigtriangledown \theta(x,y)|)}{|\bigtriangledown \theta(x,y)|} \bigtriangledown \theta(x,y)] = 0 \qquad (2)$$

in which $(i,j) \in \Omega$. Its discretized version is,

$$\sum_{(m,n)\in N(i,j)} w(m,n)\sin(\theta(m,n) - \theta(i,j)) = 0 \qquad (3)$$

again with $(i,j) \in \Omega$. Expanding (3), we obtain,

$$\theta(i,j) = \arctan(\frac{\sum_{(m,n)\in N(i,j)} w(m,n)\sin\theta(m,n)}{\sum_{(m,n)\in N(i,j)} w(m,n)\cos\theta(m,n)}) + 2k\pi,$$

for $(i,j) \in \Omega$, from which the orientation estimation problem can be solved as,

$\sum_{(m,n)\in N(i,j)}[w_x(m,n) - w_x(i,j)] = 0$
$\sum_{(m,n)\in N(i,j)}[w_y(m,n) - w_y(i,j)] = 0$
$\tan[\theta(i,j)] = w_y(i,j)/w_x(i,j) \qquad (i,j) \in \Omega$

with the constraints,

$$w_x(i,j) = -u_y(i,j), \;\; w_y(i,j) = u_x(i,j), \;\; (i,j) \in B.$$

2.2. Stage 2: Intensity diffusion

In the second stage, optimization is performed such that the intensities of the pixels in the reconstructed areas are maximally smooth in the sense of

$$\widehat{u} = \min_{u}(\int_x \int_y \rho(|\bigtriangledown u(x,y|)dxdy)$$

with the constraints,

$$u(x,y) = u'(x,y) \qquad (x,y) \in B \qquad (4)$$

$$\theta(x,y) = \widehat{\theta}(x,y) \qquad (x,y) \in \Omega$$

where $u'(x,y)$ represents the correctly reconstructed image pixels at the boundaries of lost blocks; Ω is the pixel set over which reconstruction is still necessary; and $\rho(\cdot)$ is a non-negative monotonic function.

With the orientation estimates $\widehat{\theta}(x,y)$ from Stage 1, the second constraint may be refined to allow the intensities to be tagged to the orientations via the gradient direction constraint: $\frac{-u_y}{\sqrt{u_x^2+u_y^2}} = \cos[\hat{\theta}(x,y)]$ and $\frac{u_x}{\sqrt{u_x^2+u_y^2}} = \sin[\hat{\theta}(x,y)]$. This leads to an equation linking the intensities and the estimated orientations:

$$u_{xx}\cos^2\hat{\theta} + u_{yy}\sin^2\hat{\theta} - (u_{xy} + u_{yx})\cos\hat{\theta}\sin\hat{\theta} = 0 \quad (5)$$

which may be discretized into a numerical one-pass scheme based on linear equations for the estimation of $u(x,y)$ on a digital image grid.

We discretize (5) with the following scheme. For simplicity we use $u(m,n)$ to denotes $u(m\Delta, n\Delta)$.

$u_{xx}(m\Delta,n\Delta) \simeq \frac{1}{2}[u(m+1,n)+u(m-1,n)-u(m,n+1)-u(m,n-1)]/\Delta^2 + \frac{1}{4}[u(m+1,n+1)+u(m-1,n-1)+u(m+1,n-1)+u(m+1,n-1)]/\Delta^2 - u(m,n)/\Delta^2$

$u_{yy}(m\Delta,n\Delta) \simeq -\frac{1}{2}[u(m+1,n)+u(m-1,n)-u(m,n+1)-u(m,n-1)]/\Delta^2 + \frac{1}{4}[u(m+1,n+1)+u(m-1,n-1)+u(m+1,n-1)+u(m+1,n-1)]/\Delta^2 - u(m,n)/\Delta^2$

$u_{xy}(m\Delta,n\Delta) + u_{yx}(m\Delta,n\Delta) \simeq \frac{1}{2}[u(m+1,n+1)+u(m-1,n-1)-u(m-1,n+1)-u(m+1,n-1)]/\Delta^2$

Taking these equations into (5) and solving for $u(m,n)$, we obtain,

$u(m,n) = \frac{1}{4}[u(m+1,n+1)+u(m-1,n+1)+u(m+1,n-1)+u(m-1,n-1)] + \frac{1}{2}[u(m+1,n)+u(m-1,n)-$

$u(m,n+1)-u(m,n-1)]\cos 2\widehat{\theta}+\frac{1}{4}[u(m+1,n+1)+u(m-1,n-1)-u(m+1,n-1)-u(m-1,n+1)]\sin 2\widehat{\theta}$

In the case of discrete block loss (which means that each lost block has four correctly decoded neighbouring blocks), we can reconstruct the lost blocks based on the following scheme. Let

$$c_{11}^k=\begin{cases}1+\frac{1}{4}\sin 2\widehat{\theta}(m,n) & 1<n\le L,\ 1<m\le L\\ 0 & \text{otherwise}\end{cases}$$

$$c_{12}^k=\begin{cases}-\frac{1}{2}\cos 2\widehat{\theta}(m,n) & 1<m\le L\\ 0 & m=1\end{cases}$$

$$c_{13}^k=\begin{cases}1-\frac{1}{4}\sin 2\widehat{\theta}(m,n) & 1\le n<L,\ 1<m\le L\\ 0 & \text{otherwise}\end{cases}$$

$$c_{21}^k=\begin{cases}\frac{1}{2}\cos 2\widehat{\theta}(m,n) & 1<n\le L,\\ 0 & n=1\end{cases}$$

$$c_{22}^k=-1$$

$$c_{23}^k=\begin{cases}\frac{1}{2}\cos 2\widehat{\theta}(m,n) & 1\le n<L,\\ 0 & n=L\end{cases}$$

$$c_{31}^k=\begin{cases}1-\frac{1}{4}\sin 2\widehat{\theta}(m,n) & 1<n\le L,\ 1\le m<L\\ 0 & \text{otherwise}\end{cases}$$

$$c_{32}^k=\begin{cases}-\frac{1}{2}\cos 2\widehat{\theta}(m,n) & 1\le n<L,\\ 0 & n=L\end{cases}$$

$$c_{33}^k=\begin{cases}1+\frac{1}{4}\cos 2\widehat{\theta}(m,n) & 1\le n<L, 1\le m<L\\ 0 & \text{otherwise}\end{cases}$$

where $k=(m-1)L+n$, m and n are the corresponding row and column of the image block matrix, and L is the image block size. Then construct matrix A as

$$A(k,p)=\begin{cases}c_{ij}^k & p=(m-1+i-2)L+n+j-2\\ & 1\le p\le L^2, 1\le i,j\le 3\\ 0 & \text{otherwise}\end{cases}$$

and let $v[k]=-\sum_{ij}c_{ij}^k b_{ij}^k$, $k=$ 1,2,..., L^2 and $i,j=1,2,3$, where,

$$b_{ij}^k=\begin{cases}u(m+i-2,n+j-2) & 1\le m+i-2\le L\\ & 1\le n+j-2\le L\\ 0 & \text{otherwise}\end{cases}$$

Then, we can solve for the reconstructed block (in its raster vector representation) by the matrix equation

$$\hat{I}=A^{-1}v$$

3. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed algorithm, we have constructed test images such as the one shown in Figure 1. In these images, blocks of size 8×8 have been lost in a regular pattern; the black portions of the image are lost data. In our error concealment experiment, we assume high rates of block loss but also assume that the block loss is discrete; that is each lost block has four correctly decoded neighbouring blocks.

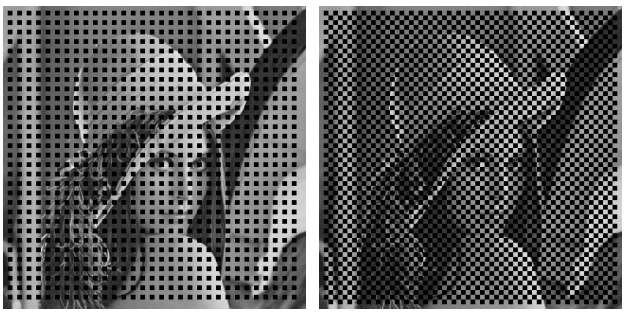
We first compare the PSNR performance of our overall error concealment scheme, using two different orientation diffusion schemes in Stage 1: one directly taken from Perona [9] and the other the modified (i.e. weighted) method described in this paper. On the 25% block loss rate (BLR) image of Figure 1(a), we obtain the following PSNR reconstruction results:

BLR	Orientation Model of [9](PSNR)	Proposed Model (PSNR)
25%	34.54	35.22

The modified orientation diffusion scheme is superior in PSNR to Perona's orientation diffusion scheme because the modified model weights image edge features in an improved manner. The error concealment result for the Lenna image with 50% discrete block loss is 31.87 in PSNR. In this case, the pixels at the four corners of the lost blocks are recovered by the interpolation of the adjacent pixels in the available image blocks. Experimental results from other images yield similar patterens of PSNR; for example, the results for the images Elaine and Boat with block loss rates of 25% and 50% are shown in the following table:

Discrete block loss rate	25%	50%
Elaine (PSNR)	35.75	32.64
Boat (PSNR)	31.31	28.33

From these experiments, we see that Elaine is similar to Lenna in PSNR. For the image Boat the PSNR drops a little because this image contains a lot of complex linear structures which are difficult to recover. Finally, from the reconstructed images in Figure 2 it is evident that the incorporation of orientation diffusion allows the visually satisfactory recovery of most blocks despite high block loss rates. Problems occur only in blocks in which orientation singularities are manifest; however these are areas of great difficulty for any error concealment scheme.



(a) (b)

Figure 1: Images for error concealment testing. (a) Image of Lena with 25% discrete block loss. (b) Image of Lena with 50% discrete block loss.

(a)

(b)

Figure 2: Error concealment results for images of Figure 1 by the proposed algorithm: (a) For Lena with 25% discrete block loss. PSNR is 35.22. (b) For Lena with 50% discrete block loss. PSNR is 31.87.

4. CONCLUSIONS

In this paper, we present a novel diffusion based error concealment scheme. We adopt an improved orientation estimation model based on Perona's orientation diffusion [9]. This model is found to be able to reconstruct complex orientation fields in lost image blocks. Based on the orientation field estimated, an efficient directional error concealment scheme is presented. This scheme is shown to be well-posed and can be solved using linear equations. Experiments for images such as Lena show that our model can give better results than more traditional models, and without an increased computational cost.

5. REFERENCES

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